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RESEARCH REPORT No. EM-169

A Convergent 'Farfield' Expansion for Two-Dimensional Radiation Functions

S. N. KARP

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A CONVERGENT 'FARFIELD' EXPANSION FOR TWO-DIMENSIONAL RADIATION FUNCTIONS

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Project Director

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Abstract

Radiative solutions of the three-dimensional Helmholtz equation $\Delta u + k^2 u = 0$ are known to be representable in the form $u = r^{-1} \exp(ikr) f(1/r,\theta,\phi)$ with $f(1/r,\theta,\phi) = f_o(\theta) + \sum_{n=1}^{\infty} f_n(\theta,0) r^{-n}$, where the series is both asymptotic and convergent. In fact $f(1/r,\theta,\phi)$ is known to be an analytic function of $1/r$ for $|r| > a$ provided all the singularities of u are contained in the sphere of radius a. On the other hand, in two dimensions, the corresponding farfield expansion

$$u \cong r^{-1/2} e^{ikr} \left[f_o(\theta) + \sum_{n=1}^{\infty} f_n(\theta) r^{-n} \right]$$

is asymptotic, but is not convergent. We remedy this defect by showing that any radiative solution of the two-dimensional Helmholtz equations can be written in the form $u = H_o^{(1)}(kr)\, E(\frac{1}{r},\theta) + H_1^{(1)}(kr) O(\frac{1}{r},\theta)$, where the functions E and O are analytic functions of $\frac{1}{r}$, for $|r| > a$, and are of course expressible as convergent power series in $\frac{1}{r}$. Here a is the radius of the smallest circle containing the singularities of u. Furthermore, we show how to deduce $E(0,\theta)$ and $O(0,\theta)$ from the customary farfield datum, namely $f_o(\theta)$, and vice versa.

The convergence of the new formula may be useful in converting indoor measurements into farfield patterns.

Table of Contents

		Page
I.	INTRODUCTION	1
II.	ANALYSIS	5
III.	DETERMINATION OF THE COEFFICIENTS	8
	BIBLIOGRAPHY	12

ERRATA

EM-169

S. N. Karp

A CONVERGENT 'FARFIELD' EXPANSION FOR TWO-DIMENSIONAL RADIATION FUNCTIONS

page 5, line 1 - Replace $H_n(kn)$ with $H_n(kr)$

page 5, equation (7) - Replace $n > a$, with $r > a$,

page 5, line immediately following equation (8) - Replace $r \geq b > 0$, with $r \geq b > a > 0$,

page 6, equation (14) - Equation should read:

$$(14) \quad E(1/kb \exp i\alpha, \theta) = \sum_{n=0}^{\infty} \frac{u_n(b)\cos(n\theta+\gamma_n)\left(\frac{n}{n!}\right)\left(\frac{kb}{2}\right)^n e^{in\alpha}}{\frac{n}{n!}\left(\frac{kb}{2}\exp i\alpha\right)^n H_n(kb)} \cdot \left[P_n(1/kbe^{i\alpha})\right]$$

page 7, equation (16) - Replace $= J_{-1}(kbc^{i\alpha})$ with $= J_{-1}(kbe^{i\alpha})$

and replace $= J_o(kbc^{i\alpha})$ with $= J_o(kbe^{i\alpha})$

I. INTRODUCTION

It is a known property[1] of the Helmholtz equation, $\nabla^2 u + k^2 u = 0$, that if a solution $u(x,y,z)$ obeys the Sommerfeld radiation condition and is regular in the exterior of a sphere of radius a, then it can be expanded in the form

$$(1) \qquad u = \frac{e^{ikr}}{r} \sum_{n=0}^{\infty} \frac{f_n(\theta,\phi)}{r^n} \quad : \quad r > a ,$$

where r, θ, ϕ are polar coordinates. The series is actually convergent, and represents an analytic function of $(1/r)$. The functions f_n can be obtained recursively[2] if the complex radiation pattern $f_o(\theta,\phi)$ is given.

On the other hand, a corresponding theorem in two dimensions is unknown. To be sure, an expansion analogous to that of (1) may be constructed[3] in the form

$$(2) \qquad u(x,y) \sim \sqrt{\frac{2}{\pi kr}}\; e^{i(kr - \frac{\pi}{4})} \sum_{n=0}^{\infty} \frac{f_n(\theta)}{r^n} \quad : \quad r > a .$$

But the example of the Hankel function $H_o^{(1)}(kr)$, which is an exact solution of the two-dimensional equation, shows that a series of type (2) diverges. An attempt to repair this defect by assuming

$$(3) \qquad u \sim H_o^{(1)}(kr) \sum_{0}^{\infty} \frac{g_n(\theta)}{r^n} \quad : \quad r > a ,$$

must also fail, since the wave function $H_1^{(1)}(kr)\exp(i\theta)$ cannot be so expressed. Thus, the two-dimensional theory lags behind the three-dimensional theory, at present. The purpose of this paper is to remedy this defect.

The principal result demonstrated below is the following:

Theorem I: *Let $u(x,y) = \tilde{u}(r,\theta)$ be an outgoing wave function regular for $r > a > 0$. Then there exists a convergent expansion, valid for $r > a$, in the form*

$$(4) \qquad u = H_0^{(1)}(kr) \sum_0^\infty \frac{F_n(\theta)}{r^n} + H_1^{(1)}(kr) \sum_0^\infty \frac{G_n(\theta)}{r^n} ,$$

where the series converge uniformly and absolutely for $r \geq b > a$, and may be differentiated termwise with respect to r as often as desired.

We also prove

Theorem II: Let $E(1/r,\theta) = \sum_0^\infty r^{-n} F_n(\theta)$, $O(1/r,\theta) \equiv \sum_0^\infty r^{-n} G_n(\theta)$ *in the notation of Theorem I. Then E and O are analytic functions of $(1/r)$, and are regular in the interior of the circle* $|1/r| \leq 1/b < 1/a$ *in the complex $(1/r)$ plane, where b is any positive number greater than a. We have*

$$(4a) \qquad u = H_0^{(1)}(kr) \; E(1/r,\theta) + H_1^{(1)}(kr) \cdot O(1/r,\theta) .$$

In addition, we show how the function $F_0(\theta)$ and $G_0(\theta)$ are related to $f_0(\theta)$, the complex radiation pattern. In fact, we prove

Theorem III: The leading terms of E, O, are related to the complex radiation pattern by the formulae

$$F_0(\theta) = \frac{1}{2}\left[f_0(\theta) + f_0(\theta + \pi)\right] \tag{4b}$$

$$-iG_0(\theta) = \frac{1}{2}\left[f_0(\theta) - f_0(\theta + \pi)\right]$$

Since the functions $F_n(\theta)$, $G_n(\theta)$ may be obtained recursively, as explained later, in terms of F_0 and G_0, Theorem III makes it possible to construct the convergent expansion (4) in terms of the experimentally meaningful leading term, $f_0(\theta)$, of the asymptotic expansion (2).

It is natural to ask why the two-dimensional expansion theorem should turn out to have a more complicated form than does the three-dimensional one. The answer is that the situation in two dimensions is the more characteristic one. In fact, it can be shown that in three dimensions we have the convergent expansion

$$u = \frac{e^{ikr}}{r}\sum_0^\infty \frac{\tilde{F}_n(\theta,\phi)}{r^n} - \left(\frac{e^{ikr}}{r}\right)\sum_0^\infty \frac{G_n(\theta,\phi)}{r^n} \tag{5}$$

This expansion is completely analogous to (4), since $[(\exp ikr)/r]$ and $H_0(kr)$ are the fundamental solutions in three- and two-dimensions and since $H_1^{(1)}(kr)$ is the derivative of $H_0(kr)$. The expansion (5) can also be rewritten in the form (1) due to the fact that $[(\exp ikr)/r]' = (\exp ikr)\,[ik - 1/r)]$.

The relationships discussed in the last paragraph can be made understandable when the respective methods of proof are explained. Consider first the proof of (1). That proof* is based on the expression of u by means of Green's Theorem, using the fundamental function $((\exp ikR)/R)$ and its normal derivative on the sphere $r = a$, where R is the distance between the field point $P(r,\theta,\phi)$ and the point $P(a,\theta_o,\phi_o)$. Here P_o is a point on the sphere $r = a$. This yields an expression of u in terms of integrals involving u and its normal derivative on the boundary. The elementary functions of R which occur in the integrand can then be expanded in inverse powers of r, to prove the theorem. [For example, we have

$$\exp ikR/R = \left\{\frac{(\exp ikr)}{r}\right\}\left\{\frac{r \exp ik\,(R-r)}{R}\right\}$$

and the second factor is an analytic function of $\frac{1}{r}$ in the neighborhood of infinity.] The proof of (1) thus depends on the elementary character of the exponential function. This method doesn't seem to be applicable in two dimensions. The method of proof we have employed in the present work is quite different. It is based on the fact that the function $u(x,y)$ can be written in the form

$$u = \sum H_n(kr)\,(a_n \cos nb + b_n \sin n\,\theta) \quad ,$$

for $r > a$, and, in addition, on the well known fact that the recurrence formula for cylinder functions allows us to express $H_n(kn)$ as a linear combination of $H_0(kr)$ and $H_1(kr)$ with coefficients which are polynomials in $(1/r)$. The polynomials are Lommel's polynomials[4]. We show below that the resulting expression, namely

---

*cf. [1]

$$(6) \qquad u = H_0(kr) \sum P_n\left(\frac{1}{kr}\right) (a_n \cos n\theta + b_n \sin n\theta)$$

$$+ H_1(kr) \sum Q_n\left(\frac{1}{kr}\right) (a_n \cos n\theta + b_n \sin n\theta) \quad ; r \geq b > a,$$

can be rearranged so as to take the form given in (4).

II. ANALYSIS

Suppose

$$(7) \qquad u_{xx} + u_{yy} + k^2 u = 0 , \qquad n > a, \quad k \text{ real},$$

$$(8) \qquad \lim_{r \to \infty} \sqrt{r}\,(u_r - iku) = 0 .$$

The function u is analytic, for $r \geq b > 0$, and hence we may write

$$(9) \qquad u = \sum_{n=0}^{\infty} \frac{H_n^{(1)}(kr)}{H_n^{(1)}(kb)} \left[u_n(b) \cos (n\theta + \gamma_n(b)\right], \; r \geq b > a > 0 ,$$

$$(10) \qquad u(b \cos \theta, b \sin \theta) \equiv \tilde{u}(b,\theta) = \sum_{n=0}^{\infty} u_n(b) \cos(n\theta + \gamma_n) ,$$

where the Fourier coefficients $u_n(b) \cos \gamma_n$, $u_n(b) \sin \gamma_n$ tend to zero more rapidly than any inverse power of n. On the other hand, we know that [4]

$$(11) \qquad H_n(kr) = H_o(kr)\, P_n\left(\frac{1}{kr}\right) + H_1(kr)\, Q_n\left(\frac{1}{kr}\right)$$

where $P_n\left(\frac{1}{kr}\right)$, $Q_n\left(\frac{1}{kr}\right)$ are the Lommel polynomials [4] of degree n in the variable (1/kr). In the notation of Watson, we have

$$(12) \qquad P_n\left(\frac{1}{r}\right) = R_{n,o}(r)$$

$$Q_n\left(\frac{1}{r}\right) = R_{n-1,1}(r)$$

Consider now the series

$$(13a) \qquad E(w,\theta) = \sum_0^\infty \frac{u_n(b) \cos (n\theta + \gamma_n)}{H_n(kb)} P_n(w)$$

$$(13b) \qquad Q(w,\theta) = \sum_0^\infty \frac{u_n(b) \cos (n\theta + \gamma_n)}{H_n(kb)} Q_n(w)$$

where w is a complex variable. Replace w by (1/kb exp iα) where $0 \leq \alpha \leq 2\pi$, and b is a fixed positive number greater than a. Then, for example, we can rewrite (13a) as follows:

$$(14) \qquad E(1/kb \exp i\alpha\theta) = \sum_{n=0}^\infty \frac{u_n(b)\cos(n\theta + \gamma_n) \left(\frac{n}{n!}\right)\left(\frac{kb}{2}\right)^n e^{in\alpha}}{\frac{n}{n!}\left(\frac{kb}{2} \exp i\alpha\right)^n H_n(kb)}$$

Now Hurwitz has shown [4] that

$$(15) \qquad \lim_{m \to \infty} \frac{\left(\frac{z}{2}\right)^{\nu+m} R_{m,\nu+1}(z)}{\Gamma\,(\nu+m+1)} = J_{\nu}(z)$$

Let $z = kb \exp i\alpha = 1/w$. Hence, by (12) we have

$$(16) \qquad \operatorname{Lim}_{n \to \infty} \frac{nP_n(1/kbe^{i\alpha}) \cdot \left(\frac{kbe^{i\alpha}}{2}\right)^{n-1}}{n!} = J_{-1}(kbc^{i\alpha})$$

$$\operatorname{Lim}_{n \to \infty} \frac{nQ_n(1/kbe^{i\alpha}) \left(\frac{kbe^{i\alpha}}{2}\right)^{n-1}}{n!} = J_o(kbc^{i\alpha})$$

On the other hand, for real b, we know [5] that

$$(17) \qquad \lim_{n \to \infty} \frac{n\left(\frac{kb}{2}\right)^n}{n!} H_n(kb) = -\frac{i}{\pi}$$

and we recall that

$$(18) \qquad |u_n(b)\cos(n\theta + \gamma_n)\; e^{-in\alpha}| < O\left(\frac{1}{n^p}\right)$$

for arbitrary p. These considerations show that the series on the right side of (14) is uniformly and absolutely convergent. Now the series (13a) is a series of polynomials in w, which, as we have just seen, converges uniformly and absolutely for $|w| = 1/kb$. Hence that series converges for $|w| < 1/kb$ and defines an analytic function. Thus the right side of the first equation of (13) is convergent for $|w| < \frac{1}{kb}$. The same is easily seen to be true of the second equation of (13). Therefore using (9) and (11) we deduce

Lemma I: If $u(r,\theta)$ is an outgoing wave function in two variables, regular outside a circle $r = a$, then

$$(19) \qquad u = H_0(kr) \sum_{n=0}^{\infty} \frac{u_n(b)\cos(n\theta+\gamma_n)}{H_n(kb)} R_{n,0}(kr)$$

$$+ H_1(kr) \sum_{n=0}^{\infty} \frac{u_n(b)\cos(n\theta+\gamma_n)}{H_n(kb)} R_{n-1,1}(kr)$$

where $R_{n,0}$ and $R_{n-1,1}$ are polynomials of degree n in the variable $(1/kr)$, and where the series converge uniformly and absolutely for $r \geq b > a$. The series also converge uniformly and absolutely for a complex r, if $|r| \geq b > a$.

This Lemma easily implies Theorem II of the Introduction, since the uniform convergence of the series for E and O, of (13a) and (13b) shows that E and O are analytic functions of $(1/r)$ in the circle $(1/r) < 1/b$ in the complex $1/r$ plane. Theorem I is a trivial corollary of Theorem II.

III. DETERMINATION OF THE COEFFICIENTS

We have thus far demonstrated the existence of a convergent expansion of the form given in equation (4). It is now appropriate to show how to determine the coefficients in this expansion. We consider two cases: In the first case u is given on some circle $r = b > a$.

In the second case the complex far field amplitude $f_o(\theta)$ is given.

The first case is trivial. In fact, the series (9) will supply the coefficient $u_n(b)$ and the phase angles γ_n. These can be inserted in (19) and the resulting series can be rearranged in descending powers of r, by the Weierstrass double-series theorem.

It is of greater interest to construct the series (4) when we are given data at $r = \infty$. Suppose therefore that we are given the complex far field amplitude $f_o(\theta)$, where, by (2)

$$(20) \qquad \lim_{r \to \infty} \sqrt{\frac{\pi kr}{2}}\, u \exp(-ikr + \frac{i\pi}{4}) = f_o(\theta)$$

Notice that $F_o(\theta) = E(0,\theta)$ and $G_o(\theta) = O(0,\theta)$. It is easily seen that, if the functions $F_o(\theta)$, $G_o(\theta)$ of equation (14) are given, then F_n and G_n can be computed recursively. To see this fact, simply substitute (4) into the equation $\nabla^2 u + k^2 u = 0$, and express all derivatives of H_o and H_1 in terms of H_o and H_1, by means of the recurrence formulae,

$$(21) \qquad H_o' = -H_1 \,, \qquad H_o'' = +\frac{1}{r} H_1 - H_o$$

$$\frac{2H_1'}{r} = \frac{-2H_o''}{r} \,, \quad H_1'' = -(\frac{1}{r} H_1 - H_o)'$$

Then collect the terms multiplying H_o and H_1. The net coefficients of H_o and H_1 must vanish identically, and this gives expressions for F_n, G_n in terms of F_o and G_o. For example we find

(22)
$$F_1 = -\frac{1}{2}(G_o'' + G_o)$$
$$G_1 = \frac{1}{2} F_o'' .$$

Therefore the problem is really to find F_o and G_o, given $f_o(\theta)$. In order to find F_o and G_o, i.e., to deduce the result given in Theorem III, we notice first that, as $r \to \infty$,

(21a) $$H_n^{(1)} \sim (-i)^n \tilde{H}_o^{(1)}$$

(21b) $$H_n^{(2)} \sim (+i)^n \tilde{H}_o^{(2)} ,$$

where

(21c) $$\tilde{H}_o^{(1)} \equiv \sqrt{\frac{2}{\pi kr}}\; e^{i(kr - \frac{\pi}{4})} ; \; \tilde{H}_o^{(2)} = \sqrt{\frac{2}{\pi kr}}\; e^{-i(kr - \frac{\pi}{4})}$$

Also, we have

(22) $$H_o^{(m)}(kr)P_n + H_1^{(m)}(kr)Q_n = H_n^{(m)}(kr) , \quad m = 1,2.$$

These equations show that as $r \to \infty$

(23) $$P_n - iQ_n \to (-i)^n ; \; P_n + iQ_n \to (+i)^n ,$$

and, consequently, that

$$(24)\qquad \left.\begin{array}{lcl} P_{2p} \to (-1)^p & : & P_{2p+1} \to 0 \\ \\ Q_{2p} \to 0 & : & Q_{2p+1} \to (-1)^p \end{array}\right\} \quad \text{for } r \to \infty$$

Therefore, by (19) and (21a) and (22), we have

$$(25)\qquad f_o(\theta) = \sum_{n=0}^{\infty} \frac{u_n(b)(-i)^n}{H_n^{(1)}(kb)} \cos(n\theta + \gamma_n).$$

But, since $F_o(\theta) = E(0,\theta)$, $G_o(\theta) = O(0,\theta)$, we find, by letting $r \to \infty$ in (4a) and by using (21a) and (4), that

$$(26)\qquad f_o(\theta) = F_o(\theta) - iG_o(\theta) = E(0,\theta) - iO(0,\theta)$$

Furthermore, if we use (19) and (12), and exploit the simplification introduced by (24) , we find that $F_o(\theta)$ contains only even-numbered Fourier coefficients, and $G_o(\theta)$ contains only odd ones. In fact

$$(27)\qquad F_o(\theta) = \sum \frac{u_{2p}(b)(-1)^p}{H_{2p}(kb)} \cos\,(2p\theta + \gamma_{2p})$$

$$G_o(\theta) = \sum \frac{u^{(b)}_{2p+1}\ (-1)^p}{H_{2p+1}(kb)} \cos\left[(2p+1)\theta + \gamma_{2p+1}\right].$$

If we compare (25) with (27), we find, in view of (26), the result expressed in Theorem III of the introduction. Thus we have expressed F_o, G_o in terms of f_o, as we desired.

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